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14. ABSTRACT
In the project we have derived new results for wave propagation in random and complex media and looked at specific applications
associated with imaging and communication through a cluttered medium. The main new theoretical result is an explicit expression for the forth moment of the wave field in the scintillation regime which is relevant for instance for laser beam propagation through the turbulent atmosphere. This is important because it allows us to analyze statistical stability of imaging and communication schemes.

We have used the results on the fourth moment to analyze wavefront correction schemes and obtained novel theoretical results that characterize the performance of these. We have also analyzed precursors that emerge in random media, in the beam regime, and how the frequency contents of the source governs the evolution of these. Moreover, we have analyzed how one can use the statistics of reflected (or transmitted) signals to infer information about the microstructure of the medium through which the signals has propagated. We have also generalized the concept of a Brewster angle associated with optimal transmission to the case with a random slab.

15. SUBJECT TERMS
electromagnetics, random media, scintillation, long range propagation, turbulence, precursor, apparent attenuation

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FINAL REPORT: AFOSR PROJECT: RESEARCH TOPICS ON CLUTTERED ENVIRONMENTS INTERROGATION AND PROPAGATION

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Abstract. In the project we have derived new results for wave propagation in random and complex media and looked at specific applications associated with imaging and communication through a cluttered medium. The main new theoretical result is an explicit expression for the forth moment of the wave field in the scintillation regime which is relevant for instance for laser beam propagation through the turbulent atmosphere. This is important because it allows us to analyze *statistical stability* of imaging and communication schemes. That is, how large the fluctuations around the mean transmitted signal are and how can we actually minimize them and understand the performance of specific imaging and transmission schemes. We have in particular used this result to analyze wavefront correction schemes and obtained novel theoretical results that characterize the performance of these. We have also analyzed precursors that emerge in random media, in the transport and beam regimes. We have shown how the theory of waves in random media can explain these and how the frequency contents of the probing signals governs the evolution of the precursor. Moreover, we have analyzed how one can use the statistics of reflected (or transmitted) signals to infer information about the microstructure of the medium through which the signals has propagated. It shows that certain features, like correlation length of the propagation medium, can be inferred from (incoherent) reflections. The issue of intrinsic versus apparent attenuation is a long standing question regarding waves in complex media. In this project we have seen how a medium with long range correlated fluctuations in the index of refraction, but no attenuation, can explain the damping behavior observed in situations with cluttered media. Similarly the concept of a Brewster angle with enhanced transmission is a classic concept for electromagnetic wave propagation. We have generalized this concept by showing how a random medium can in fact be associated with a Brewster angle that may or may not coincide with the classic Brewster angle.

Key words. electromagnetics, random media, scintillation, long range propagation, turbulence, precursor, apparent attenuation.

1. SUMMARY OF RESULTS.

1.1. Summary Main Results. The main result from the theoretical perspective is the explicit expression for the 4th moment of the wave field derived in [5]. This has been a long standing open question and is important because the fourth moment of the wave field is needed to describe the performance of imaging and communication schemes. That is, in typical situations the image or signal itself is described in terms of the second moment of the wave field while its fluctuations are described by a quantity involving the fourth moment of the wave field. The signal to noise ratio depends on (the mean of) these quantities and for an imaging scheme one seeks a design so that this is large so that the fluctuations are small relative to the signal itself.

The main result from the applications view point are: (i) the novel results on the performance of adaptive optics or enhanced transmission types of techniques using the results on the fourth moment [4]. In particular these results give a novel theoretical explanation for the memory effect. By calibrating the front shaping to one "reference star" or source one can also use this front shaping for neighborhood points of the reference star. We identify explicitly what determine the size of the neighborhood;

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(ii) the novel results for precursors arising in 3D random media which generalize the previous 1D theory [9]. We show how a precursor of algebraic decay emerges in the context of long range propagation in a random medium and how to tune appropriately the frequency contents for transmission to a certain depth; (iii) the results on Brewster angle in a layered random medium [8] which generalize the notion of a Brewster angle to heterogeneous interface zones. These results show that the optimal incidence angle for maximal transmission through a slab depends on both the properties of the random medium and the parameter contrast at the boundary of the slab; (iv) the results that show how the spectrum of the incoherent backscatter from a beam can be used to estimate the microstructure statistics of the a cluttered medium [6]. For instance, by computing the cross correlations of the incoherent waves reflected from a turbulent medium with aerosols one can in principle estimate the parameters of the turbulence; (v) a characterization of the damping regimes in complex media that conforms with the regimes seen in actual complex media. The regimes can be explained in terms of the short and long range correlations in the medium fluctuations [2, 3].

We describe the results in more detail below.

1.2. Precursors in Transport and Beam Regime; Section 2 Proposal. In [9] we generalize the theory for precursors in randomly layered media to the case of a three dimensional varying microstructure. We consider beam propagation and show how the random scattering can lead to a situation where the pulse decays algebraically rather than exponentially. We consider two principal cases (i) the case where the medium fluctuations are very rapid in the propagation direction relative to the transversal direction (ii) the case with isotropic fluctuations. In both cases we find similar expressions for the mean transmitted pulse and how we can characterize its algebraic decay. The appearance of precursors in random media can be seen as an analogue of the classic O'Dohersty-Anstey theory which was developed in geophysics and describe the spreading of a pulse traveling through random layering. In the paper [9] we also derive a 3D version of the O'Doherty-Anstey theory, that is, a description of the evolution of the pulse front shape for a wave traveling in a 3D random medium.

1.3. Fundamentals of Beam Propagation and Related Approximations; Section 3.1 Proposal. When analyzing imaging and communication schemes for waves in random media it is central to describe the fluctuations of the pulse around its mean value. Typically this amounts to computation of the fourth moment of the wave field. So far the fourth moment could be described rigorously only in some special cases. In [5] we are able to derive explicit expressions for the fourth moment of the field in the case with beam propagation in the scintillation regime. In [4] we use the expression for the fourth moment to analyze the performance of wavefront-shaping techniques of adaptive optics. We can analyze for instance how the size of the spatial light modulators (SLM) affect the stability of the focusing. Moreover, how the wavefront-shaping calibrated to a specific reference point also can be used for neighborhood points and in fact enable transmission of an image and what limits the size of the neighborhood. Such effects had been observed experimentally before, while we in [4] are able to give a theoretical derivation and quantify the effects and which physical parameters they depend on. In [15] we generalize the theory for waves in random media to the so called wide angle regime. The approximation we develop here can be used to describe waves that propagate in a spherical fashion rather than as a beam. In fact the description shows that the precursor theory developed for layered media and to laser beam propagation also can be generalized to the wide angle regime.

1.4. Paraxial Approximation for General Hyperbolic Systems; Section 3.2 Proposal. In [13] we develop a paraxial approximation for general hyperbolic systems. Here we present a unified framework that allows us to derive a general vector form for the paraxial approximation. Then propagation of electromagnetic waves and elastic waves in the beam regime appear as special cases. An important aspect of these results is that they allow us to describe how the different polarizations are statistically coupled. Such a description is relevant for instance for analysis of polarimetric imaging or passive imaging techniques using vector waves.

1.5. Deep Probing in Complex Layered Media; Section 4 Proposal. The existence of a so called Brewster angle for propagation through an interface is a classic and important phenomenon in electromagnetic wave propagation. In [8] we generalize this concept to the situation with propagation through a layered random medium. In fact the random medium can be associated with a Brewster angle of maximum transmission and this may in gereral be different from the one of the interface. The optimal angle for transmission through a random slab will then depend on both the expressions for the interface Brewster angle as well as the one associated with the bulk, that is, the random medium.

1.6. Passive and Active Imaging Through Strong Clutter; Section 5.1 Proposal. In [16, 17, 18] we analyze active and passive imaging techniques where "noise" in the environment, or sources of opportunity, can be exploited for imaging purposes. The basic situation we consider is that in a random medium the backscattered energy is very noisy and partly coherent. However, by computing correlations of the backscattered energy we can get statistically stable information that can be used for imaging purposes. In particular the depth of an interface or the background velocity can be estimated based on such correlation.

1.7. Detection Through Clutter; Section 5.2 Proposal. In detection of an object in a scenery it is important to relate the detection to a measure of certainty. The precision will depend on the noise in the reflected waves. In [11, 19] we develop a theory based on random matrix theory that identifies optimal detection rules in the context of detection and a characterization of the detection, and also localization, performance.

1.8. Multiscale analysis of SAR ; Section 5.3 Proposal. In [10] we analyze synthetic aperture radar imaging when we consider propagation through the dispersive ionosphere. From first principles we derive stochastic differential equations that give the statistical characterization of the measurements. Then we use this characterization to analyze resolution and signal to noise ratio and how they are affected by the medium microstructure.

1.9. Farbric Estimation; Section 5.4 Proposal. In general for waves propagating through microstructure, say through the turbulent atmosphere, through the earth's crust or through foliage, it is not possible to image the details of the microstructure. However, if the microstructure reasonably admits modeling in terms of a stationary stochastic process or random field then one may be able to estimate some statistics of the microstructure based on the transmitted or reflected waves. In [6] we use a model with aerosols immersed in a turbulent medium. The aerosols generate the main backscattered field, and we use this backscattered field to estimate the parameters of the turbulence, in particular the strength of the fluctuations.

2. OTHER INFORMATION.

2.1. PhD students produced. Xiaolong Long '14, Research Assistant UC Irvine; Fan Wang '13, JP Morgan Chase; Deliang Yin '13, BMO Harris Bank; Keqin Gong '13, Wells Fargo; Hua Lv '12, Wells Fargo; Li Kong '12, Wells Fargo; Feiyue Di '11, HSBC;

2.2. Contacts with Industry. Julien de Rosny Institut Langevin Paris

2.3. Patents. None

3. ABSTRACTS OF REPORTS. We give below the abstracts of the papers most closely associated with the project.

Abstract Reference [1]: We consider an acoustic waveguide with random fluctuations of its sound speed profile having long-range correlations. We investigate the role played by the statistical properties of the fluctuations on the pulse propagation. In waveguides a monochromatic wave can be decomposed over propagating modes and evanescent modes. This study is based on an asymptotic analysis of a differential equation with random coefficients describing the mode-coupling mechanism. The main result is that the net effect of the random fluctuations can be explained in terms of a random travel time correction. This result corresponds to the one derived in layered media and corroborates the robustness of results derived in randomly layered media.

Abstract Reference [2]: Experiments show that waves propagating through the earth's crust experience frequency-dependent attenuation. Three regimes have been identified with specific attenuation properties: the low-, mid-, and high-frequency regimes with attenuation in general increasing with frequency. This paper shows how the observed behavior can be explained via theory for waves in random media. It considers multiple scattering of waves propagating in non-lossy one-dimensional random media with short- and/or long-range correlations. Using stochastic homogenization theory it is possible to show that pulse propagation is described by effective fractional damping exponents. The damping exponents are related to the Hurst parameters of the random media which are characteristic parameters of the correlation properties of the fluctuations of the random media. This description is the link in between the random medium properties and the observed damping behavior. In particular a simple binary medium is shown to reproduce well the experimental attenuation properties in the low-, mid-, and high-frequency regimes.

Abstract Reference [3]: Waves propagating through heterogeneous media experience scattering that can convert a coherent pulse into small incoherent fluctuations. This may appear as attenuation for the transmitted front pulse. The classic O'Doherty-Anstey theory describes such a transformation for scalar waves in finely layered media. Recent observations for seismic waves in the earth suggest that this theory can explain a significant component of seismic attenuation. An important question to answer is then how the O'Doherty-Anstey theory generalizes to seismic waves when several wave modes, possibly with the same velocity, interact. An important aspect of the O'Doherty-Anstey theory is the statistical stability property, which means that the transmitted front pulse is actually deterministic and depends

only on the statistics of the medium but not on the particular medium realization when the medium is modeled as a random process. It is shown in this paper that this property generalizes in the case of elastic waves in a nontrivial way in that the energy of the transmitted front pulse, but not the pulse shape itself, is statistically stable. This result is based on a separation of scales technique and a diffusion-approximation theorem that characterize the transmitted front pulse as the solution of a stochastic partial differential equation driven by two Brownian motions.

Abstract Reference [4]: When waves propagate through a complex or heterogeneous medium the wave field is corrupted by the heterogeneities. Such corruption limits the performance of imaging or communication schemes. One may then ask the question: is there an optimal way of encoding a signal so as to counteract the corruption by the medium? In the ideal situation the answer is given by time reversal: for a given target or focusing point, in a first step let the target emit a signal and then record the signal transmitted to the source antenna, time reverse this and use it as the source trace at the source antenna in a second step. This source will give a sharply focused wave at the target assuming a large source aperture. Here we address this scheme in the more practical situation with a limited aperture, time harmonic signal, and finite-sized elements in the source array. Central questions are then the focusing resolution and signal-to-noise ratio at the target, their dependence on the physical parameters, and the capacity to focus selectively in the neighborhood of the target point and to transmit images. Sharp results are presented for these questions.

Abstract Reference [5]: In this paper we consider the Itô-Schrödinger model for wave propagation in random media in the paraxial regime. We solve the equation for the fourth-order moment of the field in the regime where the correlation length of the medium is smaller than the initial beam width. As applications we derive the covariance function of the intensity of the transmitted beam and the variance of the smoothed Wigner transform of the transmitted field. The first application is used to explicitly quantify the scintillation of the transmitted beam and the second application to quantify the statistical stability of the Wigner transform.

Abstract Reference [6]: When waves penetrate a medium without coherent reflectors, but with some fine scale medium heterogeneities, the backscattered wave is incoherent without any specific arrival time or the like. In this paper we consider a distributed field of weak microscatterers, like aerosols in the atmosphere, which coexists with microstructured clutter in the medium, like the fluctuations of the index of refraction of the turbulent atmosphere. We analyze the Wigner transform or the angularly resolved intensity profile of the backscattered wave when the incident wave is a beam in the paraxial regime and when the Born approximation is valid for the microscatterers. An enhanced backscattering phenomenon is proved and the properties of the enhanced backscattering cone (relative amplitude and profile) are shown to depend on the statistical parameters of the microstructure, but not on the microscatterers. These results are based on a multiscale analysis of the fourth-order moment of the fundamental solution of the white-noise paraxial wave equation. They pave the way for an estimation method of the statistical parameters of the microstructure from the observation of the enhanced backscattering cone. In our scaling argument we differentiate the two important canonical scaling regimes which are the scintillation regime and the spot dancing regime.

Abstract Reference [7]: A number of recent studies discuss the phenomenon of super resolution, that is, the fact that a target can be localized with higher resolution than half a wavelength as suggested by the classical diffraction limit. Here we discuss a special type of super resolution corresponding to a high contrast in wave speed at the location of respectively the point of observation and the one of the target. We quantify the resolution achieved in this case and discuss image stability. It turns out that the image is stable with respect to measurement noise but very sensitive to medium uncertainty. The signal-to-noise ratio can in fact be significantly enhanced by exploiting resonance frequencies and we discuss this in detail, considering source as well as reflector broadband imaging.

Abstract Reference [8]: In this paper the reflection of an obliquely incident electromagnetic wave on a randomly layered multiscale half-space is analyzed. By using homogenization and diffusion approximation theorems it is possible to get a complete description of the reflectivity of the random half-space that depends on the effective reflectivity of the interface and on the random reflectivity of the bulk medium. Particular attention is devoted to the characterization of the *Brewster* anomalies that correspond to small or even zero reflectivity. It turns out that the interface reflectivity and the random medium reflectivity as functions of the incidence angle may both possess Brewster angles that minimize or even can cancel them, but these two angles are in general different.

Abstract Reference [9]: We consider scattering of a pulse propagating through a three-dimensional random media and study the shape of the pulse in the parabolic approximation. We show that, similarly to the one-dimensional O'Doherty-Anstey theory, the pulse undergoes a deterministic broadening. Its amplitude decays only algebraically and not exponentially in time, due to the signal low/midrange frequency component. We also argue that the parabolic approximation captures the front evolution (but not the signal away from the front) correctly even in the fully three-dimensional situation.

Abstract Reference [10]: We consider Synthetic Aperture Radar (SAR) image formation in the situation when the propagation medium is random and dispersive. The propagation model is the Klein-Gordon equation with a random index of refraction and a random dispersive term. We show via a multiscale analysis how the medium heterogeneities and the dispersion affect the image. In fact, in the situation with a strong source chirp signal the main effect of the medium heterogeneities is to introduce random phase distortions in the SAR data. We carry out novel scaling analysis that gives a precise characterization of this canonical phase perturbation and how it affects image resolution and stability. The main effect of the phase perturbation is to reduce the azimuthal resolution and the signal-to-noise ratio and we quantify this performance degradation.

Abstract Reference [11]: The imaging of a small reflector embedded in a medium is a central problem in sensor array imaging. The goal is to find a reflector embedded in a medium. The medium is probed by an array of sources, and the signals backscattered by the reflector are recorded by an array of receivers. The responses between all pairs of source and receiver are collected so that the available information takes the form of a response matrix. When the data are corrupted by additive measurement noise we

show how tools of random matrix theory can help to detect, localize, and characterize the reflector.

Abstract Reference [12]: We study cumulative scattering effects on wave front propagation in time dependent randomly layered media. It is well known that the wave front has a deterministic characterization in time independent media, aside from a small random shift in the travel time. That is, the pulse shape is predictable, but faded and smeared as described mathematically by a convolution kernel determined by the autocorrelation of the random fluctuations of the wave speed. The main result of this paper is the extension of the pulse stabilization results to time dependent randomly layered media. When the media change slowly, on time scales that are longer than the pulse width and the time it takes the waves to traverse a correlation length, the pulse is not affected by the time fluctuations. In rapidly changing media, where these time scales are similar, both the pulse shape and the random component of the arrival time are affected by the statistics of the time fluctuations of the wave speed. We obtain an integral equation for the wave front, that is more complicated than in time independent media, and cannot be solved analytically, in general. We also give examples of media where the equation simplifies, and the wave front can be analyzed explicitly. We illustrate with these examples how the time fluctuations feed energy into the pulse. Explicitly, we quantify the trade-off between pulse enhancement in dynamic media and pulse fading due to scattering by the random layers.

Abstract Reference [13]: In this paper we consider a general hyperbolic system with random perturbations which models wave propagation in random media. We consider the paraxial white-noise regime, which is the regime in which the propagation distance is much larger than the diameter of the input beam which is itself much larger than the typical wavelength, and in which the correlation length of the medium is of the same order as the diameter of the input beam. Using invariant imbedding and asymptotic analysis we derive the system of Itô-Schrödinger equations that govern the propagation of the waves. The general form of the diffraction operator and the covariance matrix of the Brownian fields are computed from the eigenvalues and eigenvectors of the unperturbed system and from the two-point statistics of the random fluctuations of the medium. Applications are given for acoustic, elastic, and electromagnetic waves.

Abstract Reference [14]: In this paper the white-noise paraxial wave model is considered. This model describes for instance the propagation of laser beams in the atmosphere in some typical scaling regimes. The closed-form equations for the second- and fourth-order moments of the field are solved in two particular situations. The first situation corresponds to a random medium with a transverse correlation radius smaller than the beam radius. This is the spot-dancing regime: the beam shape spreads out as in a homogeneous medium and its center is randomly shifted according to a Gaussian process whose variance grows like the third power of the propagation distance. The second situation corresponds to a plane-wave initial condition, a small amplitude for the medium fluctuations, and a large propagation distance. This is the scintillation regime: the normalized variance of the intensity converges to one exponentially with the propagation distance, corresponding to strong intensity fluctuations and in agreement with the conjecture that the statistics of the field becomes complex Gaussian.

Abstract Reference [15]: In this paper we analyze wave propagation in three-dimensional random media. We consider a source with limited spatial and temporal support that generates spherically diverging waves. The waves propagate in a random medium whose fluctuations have small amplitude and correlation radius larger than the typical wavelength but smaller than the propagation distance. In a regime of separation of scales we prove that the wave is modified in two ways by the interaction with the random medium: first, its time profile is affected by a deterministic diffusive and dispersive convolution; second the wave fronts are affected by random perturbations that can be described in terms of a Gaussian process whose amplitude is of the order of the wavelength and whose correlation radius is of the order of the correlation radius of the medium. Both effects depend on the two-point statistics of the random medium.

Abstract Reference [16]: We study scalar waves probing a heterogeneous medium whose parameters are modeled in terms of a statistically isotropic random field. The medium is terminated by an oblique interface at one end (the bottom) and pressure release type boundary conditions at the other end (the top). The tilt of the (bottom) interface is relatively small so that the dominant contributions to the wave field are confined to a paraxial tube. This study generalizes the basic formulation in terms of Itô-Schrödinger equations in a one-dimensional deterministic background, describing the macrostructure, to the one in which the background is more complicated. It provides the first step toward the analysis of scattered waves in general background media modulated by a random microstructure. We discuss in detail the enhanced backscattering phenomenon or weak localization in this setting, with a tilted interface imbedded in the random medium, and find that the backscattering cone does not depend on the tilt. We also find that the enhanced backscattering phenomenon is not affected by the replacement of a specular interface with a diffusive interface.

Abstract Reference [17]: We analyze the notion of "field-field" cross correlations associated with scattered coda waves or clutter, observed at pairwise distinct receivers, to obtain an "empirical" Green's function (EGF) with an emphasis on high-frequency body waves. The scattered waves are generated in a slab with random medium fluctuations by an incident wave packet below. Following the dyadic parabolic scaling of wave packets, and scaling the random fluctuations appropriately, we arrive at a description in terms of a system of Îto-Schrödinger diffusion models. Studying the Wigner distributions of the fields generated by these models, leads to a "blurring" transformation providing a complete characterization of the mentioned cross correlations.

Abstract Reference [18]: The cross correlations of the wave signals emitted by ambient noise sources can be used to estimate the Green's function of the wave equation in an inhomogeneous medium. In this paper we clarify the role of random scattering in the Green's function estimation in the radiative transport regime and we show how this insight can be used to estimate the velocity of propagation of a smooth background medium.

Abstract Reference [19]: The problem addressed in this paper is the combined detection and localization of a point reflector embedded in a medium by sensor array imaging when the array response matrix is measured in a noisy environment. We construct a detection test based on reverse-time migration of the array response matrix

that is the most powerful for a given false alarm rate and prove that it is more efficient than the one based on the singular value decomposition of the response matrix. Moreover, we show that reflector localization should be performed with reverse-time migration rather than any other form of weighted-subspace migration and we give the standard deviation of the localization error.

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